Lecture Guide

Math 105 - College Algebra

Chapter 8

to accompany

"College Algebra" by Julie Miller

Corresponding Lecture Videos can be found at

MATHVIDEOS.NET

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Last updated: 5/10/13

8.1 & 8.2 – Sequences and Series

Notation: $11, 14, 17, 20, 23, \cdots, 3n+8,$

In exercises 10-12, the *n*th term of a sequence is given. Write the first four terms of the sequence.

8.1 #10 $b_n = -n^3 + 5$

8.1 #12 $d_n = 64\left(-\frac{1}{4}\right)^n$

8.1 #22 The *n*th term of a sequence is given. Find the indicated term.
$d_n = 6n + 7$; find d_{204}

With **recursive** sequences, rather than finding the n^{th} term by substituting in values for n, we find each term based on the terms before it.

8.1 #30 Write the first five terms of the sequence defined recursively.

$d_1 = 30; d_n = \frac{1}{3}d_{n-1} - 1$

Write the first five terms of the sequence defined recursively. $\begin{cases} a_1 = 7 \\ a_n = 10 - a_{n-1} \end{cases}$

Write the first five terms of the sequence defined recursively. $\begin{cases} a_1 = 1 \\ a_2 = 1 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$

This is known as the Fibonacci sequence.

Factorial Notation:

Definition: $n! = n(n-1)(n-2)\cdots 1$

Evaluate: $4!$

Evaluate: $6!$

By definition, $0!$ equals _______.

8.1 #42 Evaluate: $\frac{10!}{6!\cdot 4!}$

8.1 #44 Evaluate: $\frac{(n-2)!}{n!}$

Summation (Sigma) Notation:

In exercises 52-58, find the sum.

8.1 #52 $\sum_{i=1}^{5}(2i+7)$

8.1 #56 $\sum_{n=2}^{4}\left(\frac{1}{3}\right)^n$

8.1 #58 $\sum_{i=1}^{50}4$

8.1 #76 Write using summation notation. There may be multiple representations. Use i as the index of summation.

$$3+\frac{1}{2}+\frac{5}{27}+\frac{3}{32}+\cdots+\frac{n+2}{n^3}$$

Arithmetic Sequences

Arithmetic sequences are sequences in which a common _________________________ exists between terms.

Example: 2, 4, 6, 8, ... , 200, ...

In exercises 16-18, determine whether the sequence is arithmetic. If so, find the common difference.

8.2 #16 $8, 0, -8, -16, \dots$

8.2 #18 $2, 4, 8, 16, \dots$

8.2 #22 Write the first five terms of an arthmetic sequence $\{a_n\}$, based on the given information about the sequence.

$a_1 = 6; d = 5$

Given $a_1 = 7$ and $d = 9$, find a_{48}.

8.2 #40 Find the 46th term of an arithmetic sequence with $a_1 = 210$ and $a_{60} = -262$.

8.2 #42 Find the number of terms of the finite arithmetic sequence.

$$7, 16, 25, 34, \dots, 574$$

For the arithmetic sequence with $a_5 = 47$ and $a_{11} = 113$, find a_1.

Add: $1 + 2 + 3 + \cdots + 998 + 999 + 1000$.

Given the sequence 2, 5, 8, 11, 14, ... find the following sums:

$S_1 =$

$S_2 =$

$S_3 =$

$S_4 =$

<u>Mini-Summary:</u>

$a_n = a_1 + (n-1)d$

$S_n = \frac{n}{2}(a_1 + a_n)$

Substituting the first (above) into the second give the following:

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

8.2 #50 Find the sum of the first 60 terms of the sequence. $\{2, 10, 18, 26, \ldots\}$

8.2 #58 Find the sum. $\sum_{i=1}^{40}(3i-7)$

For the given arithmetic sequence, the 82nd term, a_{82}, is equal to -370, and the 6th term, a_6, is equal to 10. Find the value of the 33rd term, a_{33}.

*Write each using sigma notation:

$87 + 92 + 97 + 102 + 107$

$172 + 168 + 164 + 160 + 156 + 152 + 148$

$$\frac{1}{8} + \frac{2}{9} + \frac{3}{10} + \frac{4}{11} + \frac{5}{12}$$

8.3 – Geometric Sequences and Series

Geometric Sequences and Series

Geometric sequences and series have a common _________________ between terms.

Formulas: $a_n = a_1 r^{n-1}$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

In exercises 18 and 24, determine whether the sequence is geometric. If so, determine the value of r.

8.3 #18 $5, -\frac{5}{3}, \frac{5}{9}, -\frac{5}{27}, \ldots$

8.3 #24 $\frac{5}{a^2}, \frac{15}{a^4}, \frac{45}{a^6}, \frac{135}{a^8}, \ldots$

8.3 #28 Write the first five terms of a geometric sequence $\{a_n\}$ based on the given information.

$a_1 = 80$ and $r = -\frac{4}{5}$

Find a_8 if $a_1 = 4$ and $r = \frac{-1}{2}$.

8.3 #36 Write a formula for the nth term of the geometric sequence.

$\frac{18}{5}, \frac{6}{5}, \frac{2}{5}, \frac{2}{15}, \ldots$

8.3 #42 Find the indicated term of a geometric sequence from the given information.

$a_1 = 16$ and $a_2 = -12$. Find the fifth term.

8.3 #54 Find a_1 and r for a geometric sequence $\{a_n\}$ from the given information.

$a_3 = 45$ and $a_6 = -\frac{243}{25}$

Find n for the geometric sequence having $a_1 = 1$, $a_n = -128$ and $r = -2$.

For exercises 58 and 62, find the sum of the geometric series, if possible.

8.3 #58 $\sum_{j=1}^{7} 2\left(\frac{3}{4}\right)^{j-1}$

8.3 #62 $4 + 12 + 36 + \cdots + 78{,}732$

The sum of an infinite geometric series is given by $S_\infty = \frac{a_1}{1-r}$, provided $|r| < 1$.

Note that if $r = 1$,
$2 + 2 + 2 + 2 + 2 + \cdots$ has no infinite sum.

We say that this series ___________________.

For exercises 66 and 70, find the sum of the geometric series, if possible.

8.3 #66 $1 + \frac{1}{6} + \frac{1}{36} + \frac{1}{216} + \cdots$

8.3 #70 $1 + 6 + 36 + 216 + \cdots$

8.3 #80 Write the repeating decimal $0.7\overline{8}$ as a fraction.

Review of sections 8.1 – 8.3

1. Given $a_1 = 62$, and $d = 7$, find a_{43} and S_{102}.

2. Find the first four terms of this recursive sequence: $\begin{cases} a_1 = 17 \\ a_n = 2a_{n-1} + 5 \end{cases}$

3. Write each of the following using sigma notation:

a. $131 + 122 + 113 + 104 + 95 + 86$

b. $1 + 8 + 27 + 64 + 125$

c. $40 + 20 + 10 + 5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8}$

d. $\frac{17}{53} + \frac{19}{50} + \frac{21}{47} + \frac{23}{44} + \frac{25}{41}$

4. Find both a recursive formula and a general formula for each sequence:

a. $4, 12, 36, 108, 324, \cdots$

b. $38, 45, 52, 59, 66, \cdots$

5. Find a_1 for the arithmetic sequence with $a_{43} = 218$ and $a_{51} = 507$.

6. Find the sum of the first one hundred even numbers.

8.4 – Proof by Induction

Principle of Mathematical Induction

Let S_n be a statement involving the positive integer n, and let k be an arbitrary positive integer. Then S_n is true for all positive integers n if

1. S_1 is true, and
2. The truth of S_k implies the truth of S_{k+1}.

<u>Steps:</u>

1. Show that the statement is true for n=1.
2. Assume the statement is true for n=k.
3. Show that the statement is true for n=k+1.

8.4 #8 Use mathematical induction to prove the given statement for all posible integers n.

$$2+8+14+\cdots+(6n-4)=n(3n-1)$$

8.4 #11 Use mathematical induction to prove the given statement for all posible integers n.

$$8+4+0+\cdots+(-4n+12)=-2n(n-5)$$

Prove: $4 + 12 + 20 + \cdots + (8n - 4) = 4n^2$

8.5 – The Binomial Theorem

<u>Pascal's Triangle:</u>

$(a + b)^0 =$

$(a + b)^1 =$

$(a + b)^2 =$

$(a + b)^3 =$

8.5 #18b Write the expansion of $(c - d)^3$.

$(2x - 3y)^5 =$

Coefficients of a Binomial Expansion

Let n and r be nonnegative integers with $n \geq r$. The expression $\binom{n}{r}$ (read as "n choose r") is called a **binomial coefficient** and is defined by

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Note: The notation $_nC_r$ is often used in place of $\binom{n}{r}$.

Evaluate: $\binom{52}{5} =$

Calculator keystrokes:

8.5 #20 Evaluate the given expression. Compare the result to the sixth row of Pascal's traingle.

a. $\binom{5}{0}$

b. $\binom{5}{1}$

c. $\binom{5}{2}$

d. $\binom{5}{3}$

e. $\binom{5}{4}$

f. $\binom{5}{5}$

The Binomial Theorem

Let n be a positive integer. The expansion of $(a+b)^n$ is given by

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$
$$= \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$$

8.5 #26 Expand using the binomial theorem.

$(5x+3)^5$

8.5 #32 Expand using the binomial theorem.

$(3y^2-z)^5$

kth Term of a Binomial Expansion

Let n and k be positive integers with $k \leq n+1$. The kth term of $(a+b)^n$ is

$$\binom{n}{k-1}a^{n-(k-1)}b^{k-1}$$

8.5 #44 Find the indicated term of the binomial expansion.

$(y^3+2z^2)^{14}$; tenth term

8.5 #50 Find the indicated term of the binomial expansion.

$(a^3+4b)^6$ Find the term containing a^9.

Chapter 8 Review

1. Expand: $(2x - 3y)^6$

2. Find the coefficient of the x^5y^{12} term of $(2x + y)^{17}$.

3. Prove by induction:

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2}$$

4. Find the sum: $40 + 30 + \frac{45}{2} + \frac{135}{8} + \cdots$

5. Find S_8 when $a_1 = 4$ and $r = \frac{1}{4}$.

6. Write each of the following using sigma notation:

a. $117 + 115 + 113 + 111 + 109$

b. $60 + 30 + 15 + 7.5 + 3.75$

7. Add to 117 terms: $18 + 23 + 28 + \cdots$

8. Given $a_{41} = 113$ and $a_{75} = 305$, find a_{211} for this arithmetic sequence.

9. For the arithmetic sequence with $a_5 = 32$ and $a_9 = 70$, find a_1.

10. Determine whether each is arithmetic or geometric. Then find a formula for the general term, a_n.

a. $5, 11, 17, 23, \ldots$

b. $4, 12, 36, 108, \ldots$

Final Exam Review

1. Solve: $x^2 - 12 \geq x$

2. Find the balance of an account after 5 years if $3600 is invested at 4.5%, compounded monthly.

3. Multiply: $\begin{bmatrix} 2 & 3 & -4 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 & -8 \\ -1 & 3 \end{bmatrix}$

4. In 2004, a company had a net worth of $43 million. In 2008, the same company had a net worth of $62 million. Letting $t = 0$ represent the year 2004, find a function for the worth of the company as a function of time.

5. Given $A = \begin{bmatrix} -3 & 5 \\ -1 & 4 \end{bmatrix}$, find A^{-1}, if it exists.

6. Solve: $|2x + 7| - 3 \geq 5$

7. List the possible rational zeroes of the function $g(x) = 9x^4 + 5x^3 - 2x^2 + 4$

8. Name all asymptotes and intercepts of $f(x) = \frac{x-3}{2x+1}$.

9. For the rational function described in exercise 8, finish the following statements:

$As\ x \to -\infty, y \to$__________

$As\ x \to +\infty, y \to$__________

10. How long would it take \$1500 to double when invested at 2.2%, compounded daily?

11. Find the vertices of the polygon formed:

$$\begin{cases} x \geq 2 \\ y \geq 1 \\ y \leq -x + 11 \\ y \geq x + 3 \end{cases}$$

12. Find S_{143} for the sequence $\frac{3}{5}, 1, \frac{7}{5}, \frac{9}{5}, \ldots$

13. Find the domain of each:

a. $f(x) = \sqrt{3x - 5}$

b. $g(x) = \frac{x+2}{3x-5}$

c. $h(x) = log_4(3x - 5)$

d. $f(x) = \sqrt{x^2 - 25}$

14. Find S_5 for the geometric sequence with $a_1 = 6$ and $r = -2$.

15. Find the sum, if possible:

$$4 + 4(-3) + 4(-3)^2 + 4(-3)^3 + \cdots$$

16. Find the focus and directrix of

$$(y + 3)^2 = -12(x + 2)$$

17. The length of a rectangle is 4 cm more than its width. Find a function which represents the rectangle's area in terms of its width.

18. Find the foci:

$$x^2 - 4y^2 - 12x - 8y + 16 = 0.$$

19. Sketch the graph of

$$y = (x+3)^2(x-4)(x-1)^4$$

20. What does Descartes' Rule of Signs tell us about the possible number of positive and negative real zeroes of p(x)?

$$p(x) = 2x^7 + x^6 + x^5 - x^4 - x^3 + x^2 - x + 2$$

21. Solve: $2^{x+3} = 17$