Lecture Guide

Math 105 - College Algebra

Chapter 6

to accompany

"College Algebra" by Julie Miller

Corresponding Lecture Videos can be found at

MATHVIDEOS.NET

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Last updated: 4/6/13

6.1 – Solving a System of Linear Equations Using Matrices

6.1 #18 Write the augmented matrix for the given system:

$$\begin{cases} 2(y-x) = 4-8x \\ \quad 5y = 6-x \end{cases}$$

<u>Allowable Row Operations:</u>

1. You may ________________________ rows.

2. You may ______________________ a row by a non-zero constant (scalar).

3. You may add a ________________________ of one row to another row.

Goal/Process:

Row-Echelon Form and Reduced Row-Echelon Form

A matrix is in **row-echelon form** if it satisfies the following conditions.

1. Any rows consisting entirely of zeros are at the bottom of the matrix.
2. For all other rows, the first nonzero entry is 1. This is called the leading 1.
3. The leading 1 in each nonzero row is to the right of the leading 1 in the row immediately above.

Note: A matrix is in **reduced row-echelon form** if it is in row-echelon form with the added condition that each row with a leading entry of 1 has zeros above the leading 1.

6.1 #26 Write a system of linear equations represented by the augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -1/2 \end{array}\right]$$

6.1 #34 Perform the elementary row operation

$R_1 \Leftrightarrow R_2$ on $\left[\begin{array}{ccc|c} 1 & 5 & 6 & 2 \\ 2 & 1 & 5 & 1 \\ 4 & -2 & -3 & 10 \end{array}\right]$.

6.1 #38 Perform the elementary row operation

$-4R_1 + R_3 \longrightarrow R_3$ on $\left[\begin{array}{ccc|c} 1 & 5 & 6 & 2 \\ 2 & 1 & 5 & 1 \\ 4 & -2 & -3 & 10 \end{array}\right]$.

6.1 #52 Solve the system using Gauss-Jordan elimination.

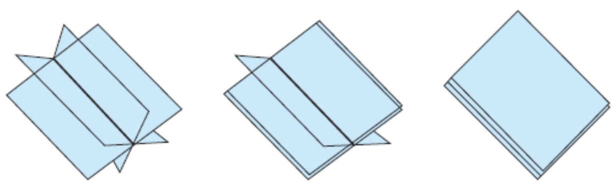
$$\begin{cases} 2(x-y) = 4x + y - 40 \\ \quad 9y = 105 - 3x \end{cases}$$

6.1 #56 Solve the system using Gauss-Jordan elimination.

$$\begin{cases} 2x - 8y + 54z = -4 \\ x - 2y + 14z = -1 \\ x - 3y + 19z = -3 \end{cases}$$

6.2 – Inconsistent Systems and Dependent Systems

A system of linear equations in three variables may have **no solution** or **infinitely many solutions**. The system has no solution if the planes are parallel and infinitely many solutions if the equations represent planes that intersect in a common line or common plane. In section 5.2, we only solved systems which had a unique (x, y, z) triple as a solution. We now turn our attention to the more "interesting" cases.



6.2 #30 Solve the system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{cases} x - 3y + 17z = 1 \\ x - y + 7z = 2 \\ 2x - 5y + 29z = 5 \end{cases}$$

6.2 #32 Solve the system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{cases} x + 3y + 9z = 12 \\ 2x + 7y + 22z = 26 \\ -5x - 17y - 53z = -64 \end{cases}$$

6.2 #36 Solve the system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{cases} 2x - y - 5z = -3 \\ x - 2y - 7z = -12 \end{cases}$$

6.2 #38 Solve the system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{cases} -x + 2y + 7z = 14 \\ 10x - 20y - 70z = -140 \\ -\frac{1}{7}x + \frac{2}{7}y + z = 2 \end{cases}$$

6.3 – Operations on Matrices

The ____________ of a matrix is its "size".

6.3 #24 a. Give the order of the matrix.
b. Classify the matrix as a square matrix, row matrix, column matrix, or none of these.

$$\begin{bmatrix} -4 & 10 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 7 & 0 & 9 & -1 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

6.3 #28 Determine the value of the element a_{23} of the matrix A.

$$A = \begin{bmatrix} 3 & -6 & \frac{1}{3} \\ 2 & 4 & 0 \\ \sqrt{5} & 11 & 8.6 \\ \frac{1}{2} & 4 & 2 \end{bmatrix}$$

For exercises 37 and 40, add or subtract the given matrices, if possible.

$$A = \begin{bmatrix} 6 & -1 \\ 7 & {}^{1}/_{2} \\ 2 & \sqrt{2} \end{bmatrix}; B = \begin{bmatrix} -9 & 2 \\ 6.2 & 2 \\ {}^{1}/_{3} & \sqrt{8} \end{bmatrix}; C = \begin{bmatrix} 11 & 4 \\ 1 & -{}^{1}/_{3} \\ 1 & 6 \end{bmatrix}$$

6.3 #37 $C - A + B$

6.3 #40 $C + D$

$$C = \begin{bmatrix} 11 & 4 \\ 1 & -\frac{1}{3} \\ 1 & 6 \end{bmatrix}; \; D = \begin{bmatrix} 2 & 3 & 8 \\ -1 & 6 & 1/6 \end{bmatrix}$$

6.3 #48 Perform the indicated row operations.

$$A = \begin{bmatrix} 2 & 4 & -9 \\ 1 & \sqrt{3} & \frac{1}{2} \end{bmatrix}; \; B = \begin{bmatrix} -1 & 0 & 4 \\ 2 & 9 & \frac{2}{3} \end{bmatrix}$$

Find $-8A - 2(A + B)$.

For exercise 54, given the matrices A and B, solve for X.

$$A = \begin{bmatrix} 1 & 6 \\ 4 & -2 \end{bmatrix}; \; B = \begin{bmatrix} 2 & -4 \\ 6 & 9 \end{bmatrix}$$

6.3 #54 $3B - A = -2X$

The product of an $(m\ x\ n)$ and an $(n\ x\ p)$ matrix is of the order ________________.

Example: $(4\ x\ 3)\cdot(3\ x\ 2) = 4\ x\ 2$

For exercises 60 and 64,

a. Find AB if possible.

b. Find BA if possible.

c. Find A^2 if possible.

6.3 #60 $A=\begin{bmatrix} 1 & -6 \\ 5 & 10 \end{bmatrix}$ and $B=\begin{bmatrix} -2 & 3 \\ 7 & -1 \end{bmatrix}$

6.3 #64 $A=\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & 7 & -1 \\ 0 & 4 & 1 \\ 0 & 3 & -6 \end{bmatrix}$

6.4 – Inverse Matrices

6.4 #12 Given $A=\begin{bmatrix} \sqrt{3} & 1 \\ \pi & 4 \end{bmatrix}$, verify that

a. $AI_n = A$ b. $I_nA = A$

6.4 #16 Determine whether A and B are inverses.

$$A=\begin{bmatrix} 4 & -1 \\ -8 & 6 \end{bmatrix};\ B=\begin{bmatrix} 3/8 & 1/16 \\ 1/2 & 1/4 \end{bmatrix}$$

There are two methods for finding inverses of matrices:

Method #1... $[A|I] \to [I|A^{-1}]$

This method <u>always</u> works.

Method #2 (only works for __________ matrices)

6.4 #22 Determine the inverse of the given matrix if possible. Otherwise, state that the matrix is singular.

$$A=\begin{bmatrix} 5 & -3 \\ 10 & -7 \end{bmatrix}$$

6.5 – Determinants and Cramer's Rule

6.5 #18 Evaluate the determinant of the matrix.

$$D = \begin{bmatrix} 8/9 & 4 \\ 5/2 & 18 \end{bmatrix}$$

Cofactor of an Element of a Matrix

Given a square matrix $A = [a_{ij}]$, the **cofactor** of a_{ij} is $(-1)^{i+j}M_{ij}$, where M_{ij} is the minor of a_{ij}.

Evaluating the Determinant of an $n \times n$ Matrix by Expanding Cofactors

Step 1 Choose any row or column.
Step 2 Multiply each element in the selected row or column by its cofactor.
Step 3 The value of the determinant is the sum of the products from step 2.

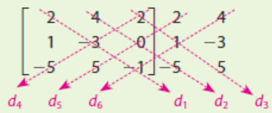
6.5 #34 Evaluate the determinant of the matrix and state whether the matrix is invertible.

$$D = \begin{bmatrix} -3 & 1 & -2 \\ 10 & 5 & 8 \\ 6 & 7 & -4 \end{bmatrix}$$

TIP The determinant of a 3×3 matrix can also be evaluated by using the "method of diagonals."

Step 1: Recopy columns 1 and 2 to the right of the matrix
Step 2: Multiply the elements on the diagonals labeled d_1 through d_6 (each diagonal has three elements).
Step 3: The value of the determinant is $(d_1 + d_2 + d_3) - (d_4 + d_5 + d_6)$.

The determinant from Example 4 is evaluated as follows:



$= [(2)(-3)(-1) + (4)(0)(-5) + (2)(1)(5)]$
$\quad - [(2)(-3)(-5) + (2)(0)(5) + (4)(1)(-1)]$
$= -10$

Some students find the method of diagonals to be a faster technique to find the determinant of a 3×3 matrix. However, it is critical to note that the method of diagonals only works for the determinant of a 3×3 matrix.

Compute the value of the determinant using this "tip".

$$D = \begin{bmatrix} -3 & 1 & -2 \\ 10 & 5 & 8 \\ 6 & 7 & -4 \end{bmatrix}$$

6.5 #44 Solve the system if possible using Cramer's Rule. If the system does not have a unique solution, give the number of solutions.

$$\begin{cases} 11x + 6y = 8 \\ 2x = 9y + 5 \end{cases}$$

6.5 #48 Solve the system if possible using Cramer's Rule. If the system does not have a unique solution, give the number of solutions.

$$\begin{cases} x = 4y + 5 \\ 3(x - 4) = 12y \end{cases}$$

6.5 #52 Solve the system if possible using Cramer's Rule. If the system does not have a unique solution, give the number of solutions.

$$\begin{cases} -5x - 6y + 8z = 1 \\ 2x + y - 4z = 5 \\ 3x - 4y - z = -2 \end{cases}$$