

Lecture Guide

Math 105 - College Algebra

Chapter 5

to accompany

“College Algebra” by Julie Miller

Corresponding Lecture Videos can be found at



Prepared by

Stephen Toner & Nichole DuBal

Victor Valley College

Last updated: 3/30/13

5.1 – Systems of Linear Equations in Two Variables and Applications

Consistent systems,

Independent equations consist of

Three Cases:

A. consistent and independent

B. inconsistent and independent

C. consistent and dependent

5.1 #8 Determine if the ordered pair is a solution to the system of equations.

$$\begin{cases} -11x + 6y = -4 \\ 7x + 3y = 23 \end{cases}$$

a. $\left(1, \frac{7}{6}\right)$

b. $(2, 3)$

5.1 #16 Solve the system of equations by using the substitution method.

$$\begin{cases} 2x + y = 2 \\ 5x + 3y = 9 \end{cases}$$

5.1 #24 Solve the system of equations by using the addition method.

$$\begin{cases} -3(x - y) = y - 14 \\ 2x + 2 = 7y \end{cases}$$

5.1 #30 Solve the system of equations by using the any method. If a system does not have one unique solution, state whether the system is inconsistent or whether the equations are dependent.

$$\begin{cases} -4x - 8y = 2 \\ 2x = 8 - 4y \end{cases}$$

5.1 #32 Solve the system of equations by using the any method. If a system does not have one unique solution, state whether the system is inconsistent or whether the equations are dependent.

$$\begin{cases} 2x - y = 8 \\ x - \frac{1}{2}y = 4 \end{cases}$$

5.1 #50 A pharmacist wants to mix a 30% saline solution with a 10% saline solution to get 200 mL of a 12% saline solution. How much of each solution should she use?

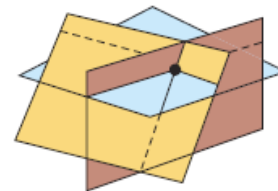
5.1 #58 The average of an electrician's hourly wage and a plumber's hourly wage is \$33. One day a contractor hires the electrician for 8 hr of work and the plumber for 5 hr of work and pays a total of \$438 in wages. Find the hourly wage for the electrician and for the plumber.

5.1 #60 A fishing boat travels along the east coast of the United States and encounters the Gulf Stream current. It travels 44 mi north with the current in 2 hr. It travels 56 mi south against the current in 4 hr. Find the speed of the current and the speed of the boat in still water.

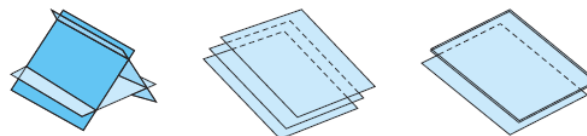
5.2 – Systems of Linear Equations in Three Variables and Applications

A solution to a system of linear equations in three variables is an ordered _____

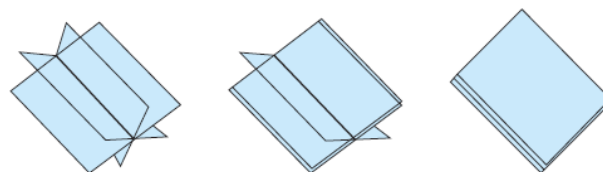
(x, y, z) that satisfies each equation in the system. Geometrically, a solution is a point of intersection of the planes represented by the equations in the system.



A system of linear equations in three variables may have **no solution**. This occurs if the equations represent planes that do not intersect. In such a case, we say that the system is _____.



A system of linear equations in three variables may also have **infinitely many solutions**. This occurs if the equations represent planes that intersect in a common line or common plane. In such a case, we say that the equations are _____.



In this section, systems will only have three possible solutions: a unique (x, y, z) triple, no solution, or infinitely many solutions. In section 6.2, we will learn how to describe linear and planar dependence algebraically.

5.2 #12 Determine if the ordered triple is a solution to the system of equations.

$$\begin{cases} 2x - 3y + z = -12 \\ x + y - 2z = 9 \\ -3x + 2y - z = 7 \end{cases}$$

a. $(2, 5, -1)$

b. $(1, 4, -2)$

For exercises 18-26, solve the system of equations. If a system does not have one unique solution, determine the number of solutions to the system.

$$5.2 \#18 \quad \begin{cases} 3x = 5y - z + 13 \\ -(x - y) - z = x - 3 \\ 5(x + y) = 3y - 3z - 4 \end{cases}$$

$$5.2 \#22 \quad \begin{cases} 4x - y + 2z = 1 \\ 3x + 5y - z = 2 \\ -9x - 15y + 3z = 0 \end{cases}$$

$$5.2 \#26 \quad \begin{cases} 3x + 2y + 5z = 6 \\ \quad \quad 3y - z = 4 \\ 3x + 17y = 26 \end{cases}$$

5.2 #38 Pierre inherited \$120,000 from his uncle and decided to invest the money. He put part of the money in a money market account that earns 2.2% simple interest. The remaining money was invested in a stock that returned 6% in the first year and a mutual fund that lost 2% in the first year. He invested \$10,000 more in the stock than in the mutual fund, and his net gain for 1 yr was \$2820. Determine the amount invested in each account.

5.3 – Partial Fraction Decomposition

For exercises 12-20, set up the form for the partial fraction decomposition. Do not solve for A , B , C , and so on.

$$5.3 \#12 \quad \frac{20x-4}{(x-5)(3x+1)}$$

$$5.3 \#18 \quad \frac{-3x^2+2x+8}{x^3+4x^2+4x}$$

$$5.3 \#20 \quad \frac{17x^2-7x+18}{7x^3+42x}$$

$$5.3 \#22 \quad \frac{3x^3-4x^2+11x-12}{x^4+6x^2+9}$$

For exercises 28-38, find the partial fraction decomposition.

$$5.3 \#28 \quad \frac{20x-4}{(x-5)(3x+1)}$$

$$5.3 \#34 \quad \frac{-3x^2+2x+8}{x^3+4x^2+4x}$$

$$5.3 \#38 \quad \frac{x^4-4x^3+11x^2-13x+12}{x^3+2x}$$

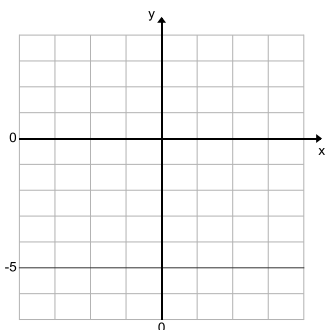
5.4 – Systems of Nonlinear Equations in Two Variables

Possible Cases:

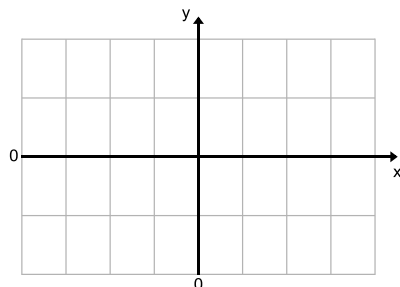
For exercises 10 and 14,

- Graph the equations in the system.
- Solve the system using the substitution method.

$$5.4 \#10 \quad \begin{cases} y = -x^2 + 3 \\ y - 2x = 0 \end{cases}$$



$$5.4 \#14 \quad \begin{cases} x^2 + y^2 = 10 \\ y = \sqrt{x-2} \end{cases}$$



5.4 #20 Solve the system using the addition method.

$$\begin{cases} 3x^2 + y^2 = 21 \\ 4x^2 - 2y^2 = -2 \end{cases}$$

5.4 #26 Solve the system using the addition method.

$$\begin{cases} 4x^2 = 4 - y^2 \\ 16y^2 = 144 + 9x^2 \end{cases}$$

5.4 #46 Find the dimensions of a rectangle whose perimeter is 56 cm and whose area is 192 cm^2 .

5.4 #48 An electronic sign for a grocery store is in the shape of a rectangle. The perimeter of the sign is 72 ft and the area is 320 ft^2 . Find the length and width of the sign.

5.5 – Inequalities and Systems of Inequalities in Two Variables

Shading rules:

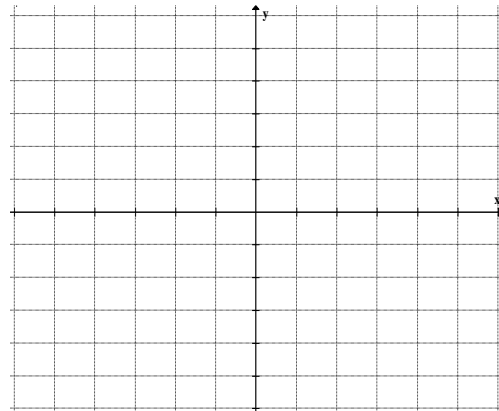
5.5 #14 Determine whether the ordered pair is a solution of the inequality $2x + 3y > 6$.

a. $(-3, 3)$

b. $(5, -1)$

c. $(0, 2)$

5.5 #18 a. Graph the solution set $2x + 5y > 10$.

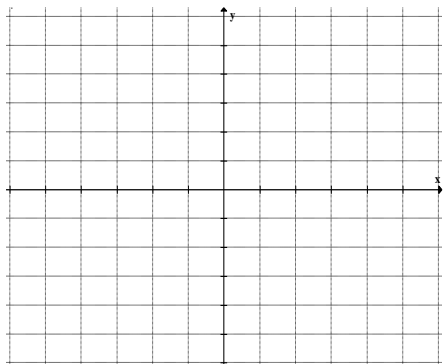


b. Explain how the graph would differ for the inequality $2x + 5y \geq 10$.

c. Explain how the graph would differ for the inequality $2x + 5y < 10$.

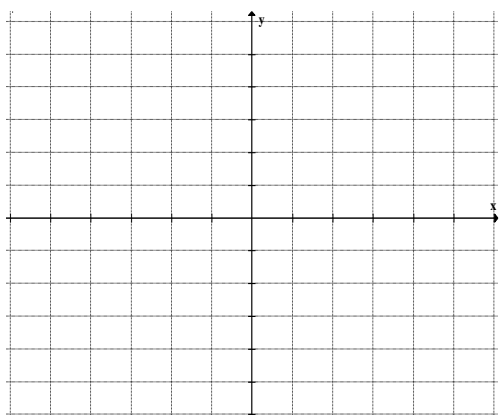
5.5 #24 Graph the solution set.

$$3x > 2y$$

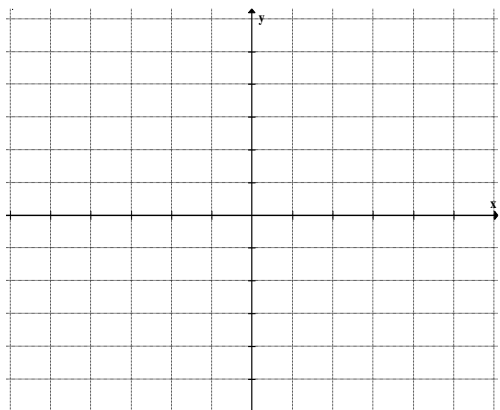


In exercises 48-58, graph the solution set. If there is no solution, indicate that the solution set is the empty set.

$$5.5 \#48 \begin{cases} y \geq \frac{1}{3}x - 2 \\ y \leq x - 4 \end{cases}$$

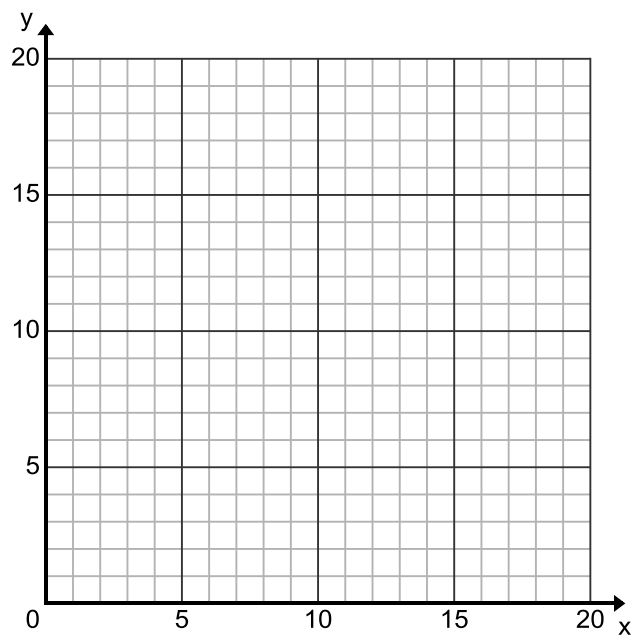


$$5.5 \#58 \begin{cases} y \leq -x^2 + 7 \\ y \leq -x + 5 \\ y > 1 \end{cases}$$



5.5 #72 Let x represent the number of country songs that Sierra puts on a playlist on her portable media player. Let y represent the number of rock songs that she puts on the playlist. For parts (a)-(e), write an inequality to represent the given statement.

- Sierra will put at least 6 country songs on the playlist.
- Sierra will put no more than 10 rock songs on the playlist.
- Sierra wants to limit the length of the playlist to at most 20 songs.
- The number of country songs cannot be negative.
- The number of rock songs cannot be negative.
- Graph the solution set to the system of inequalities from parts (a)-(e).



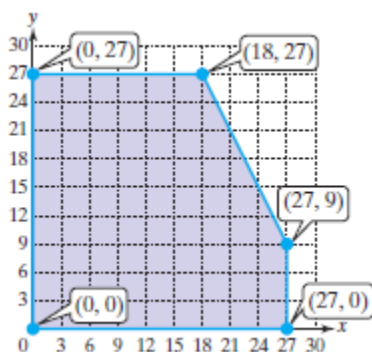
5.6 – Linear Programming

5.6 #14 An athlete burns 10 calories per minute running and 8 calories per minute lifting weights. Write the objective function $z = f(x, y)$ that represents the total number of calories burned by running for x minutes and lifting weights for y minutes.

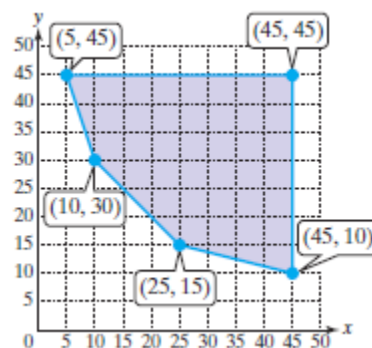
For exercises 18-19,

- Determine the values of x and y that produce the maximum or minimum value of the objective function on the given feasible region.
- Determine the maximum or minimum value of the objective function on the given feasible region.

5.6 #18 Maximize: $z = 1.8x + 2.2y$

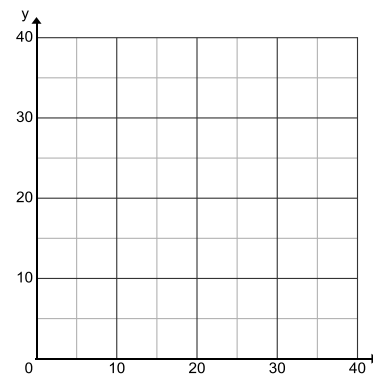


5.6 #19 Minimize: $z = 1000x + 900y$



- 5.6 #22 a. Graph the feasible region and identify the vertices.

$$\begin{cases} x \geq 0, y \geq 0 \\ 2x + y \leq 40 \\ x + 2y \leq 50 \end{cases}$$



- Determine the values of x and y that produce the maximum value of the objective function $z = 9.2x + 8.1y$ on the feasible region.
- Determine the maximum value of the objective function on the feasible region.

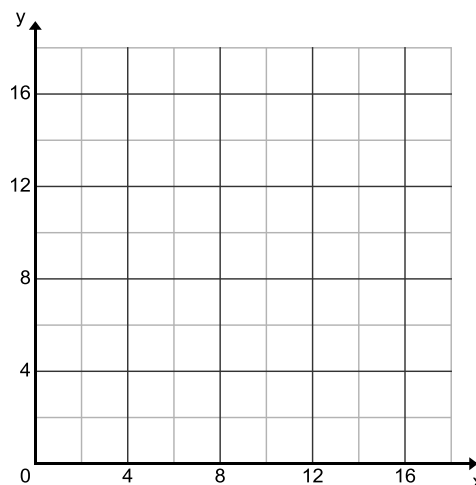
5.6 #30 Josh makes \$24/hr tutoring chemistry and \$20/hr tutoring math. Let x represent the number of hours per week he spends tutoring chemistry. Let y represent the number of hours per week he spends tutoring math.

a. Write an objective function representing his weekly income for tutoring x hours of chemistry and y hours of math.

b. The time that Josh devotes to tutoring is limited by the following constraints.

- The number of hours spent tutoring each subject cannot be negative.
- Due to the academic demands of his own classes he tutors at most 18 hr per week.
- The tutoring center requires that the tutors math at least 4 hr per week.
- The demand for math tutors is greater than the demand for chemistry tutors. Therefore, the number of hours he spends tutoring math must be at least twice the number of hours he spends tutoring chemistry.

c. Graph the system of inequalities represented by the constraints.



d. Find the vertices of the feasible region.

e. Test the objective function at each vertex.

f. How many hours tutoring math and how many hours tutoring chemistry should Josh work to maximize his income?

g. What is his maximum income?

h. Explain why Josh's maximum income is found at a point on the line $x + y = 18$.