Lecture Guide

Math 105 - College Algebra

Chapter 4

to accompany

"College Algebra" by Julie Miller

Corresponding Lecture Videos can be found at

MATHVIDEOS.NET

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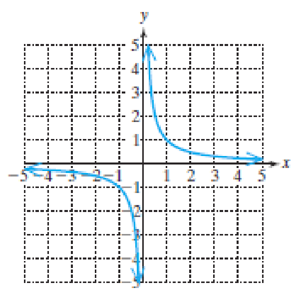
4.1 – Inverse Functions

A function is ___________________________ if it passes both a vertical and horizontal line test. If a function is one-to-one, then it is

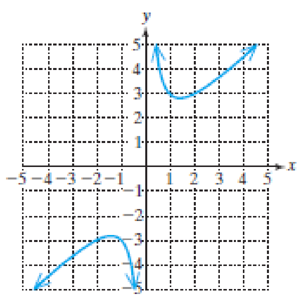
___________________________ (it has an inverse which is also a function).

In exercises 22 and 24, determine if the relation defines y as a one-to-one function of x.

4.1 #22



4.1 #24



Definition of a One-to-One Function

A function f is a **one-to-one function,** if for a and b in the domain of f, if $a \neq b$, then $f(a) \neq f(b)$, or equivalently, if $f(a) = f(b)$, then $a = b$.

4.1 #30 Use the definition of a one-to-one function to determine if $h(x) = -3x + 2$ is one-to-one.

Definition of an Inverse Function

Let f be a one-to-one function. Then g is the **inverse of** f if the following conditions are both true.

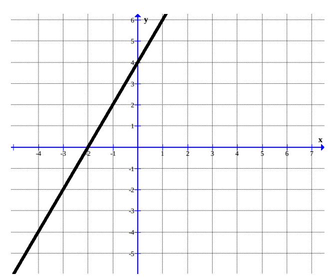
1. $(f \circ g)(x) = x$ for all x in the domain of g.
2. $(g \circ f)(x) = x$ for all x in the domain of f.

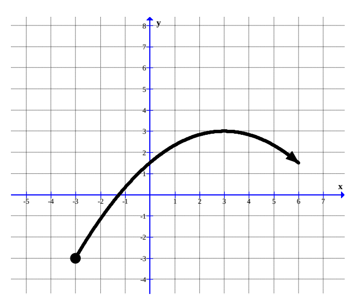
4.1 #42 Determine whether the two functions are inverses.

$w(x) = \frac{6}{x+2}$ and $z(x) = \frac{6-2x}{x}$

To find the inverse of a function from its **graph**, reflect the graph across the line ____________.
If (a, b) is on $f(x)$, then (b, a) is on the graph of its inverse.

*Given the graph of $f(x)$, graph its inverse.



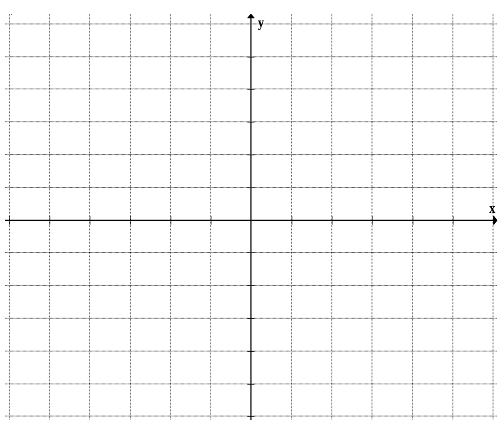


To find the inverse of a function from its **equation**, switch the x and y, and then solve for the "new" y.

In exercises 52, a one-to-one function is given. Write an equation for the inverse function.

4.1 #52 $m(x) = 2x^3 - 5$

4.1 #62 a. Graph $f(x) = \sqrt{x+2}$.



b. From the graph of f, is f a one-to-one function?

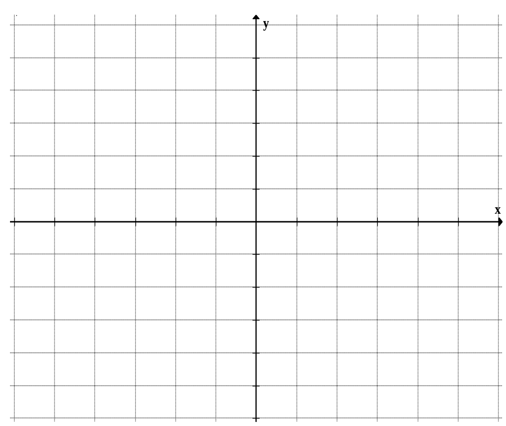
c. Write the domain of f in interval notation.

d. Write the range of f in interval notation.

e. Write an equation for $f^{-1}(x)$.

f. Explain why the restriction $x \geq 0$ is placed on f^{-1}.

g. Graph $y = f(x)$ and $y = f^{-1}(x)$ on the same coordinate system.



h. Write the domain of f^{-1} in interval notation.

i. Write the range of f^{-1} in interval notation.

In exercises 56, a one-to-one function is given. Write an equation for the inverse function.

4.1 #56 $v(x) = \frac{x-5}{x+1}$

4.1 #66 Given $f(x) = |x| - 3; x \geq 0$, write an equation for f^{-1}. (*Hint:* Sketch $f(x)$ and note the domain and range.)

4.2 & 4.3 – Exponential & Logarithmic Functions

A. Introduction

$y = a^x$ is an exponential equation.

"a" is called the ______________________.

To emphasize how exponential equations can increase rapidly, consider the following "dream" salary schedule in which a peron starts with a 2₵ salary on the first day, and every day thereafter the salary is doubled.

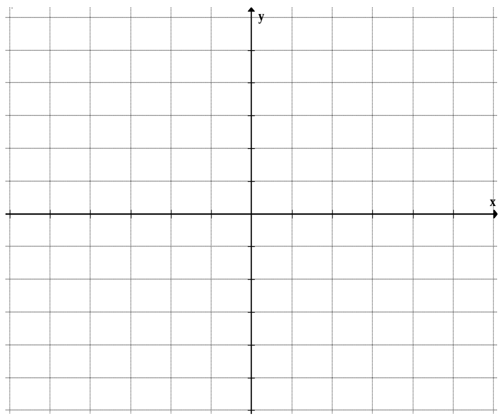
Day	Payment	Day	Payment	Day	Payment
1	2¢	11	$20.48	21	$20,971.52
2	4¢	12	$40.96	22	$41,943.04
3	8¢	13	$81.92	23	$83,886.08
4	16¢	14	$163.84	24	$167,772.16
5	32¢	15	$327.68	25	$335,554.32
6	64¢	16	$655.36	26	$671,088.64
7	$1.28	17	$1310.72	27	$1,342,177.28
8	$2.56	18	$2621.44	28	$2,684,354.56
9	$5.12	19	$5242.88	29	$5,368,709.12
10	$10.24	20	$10,485.76	30	$10,737,418.24

Avoiding Mistakes

- The base of an exponential function must not be negative to avoid situation where the function values are not real numbers. For example, $f(x) = (-4)^x$ is not defined for $x = \frac{1}{2}$ because $\sqrt{-4}$ is not a real number.
- The base of an exponential function must not equal 1 because $f(x) = 1^x = 1$ for all real numbers x. This is a constant function, not an exponential function.

B. Graphing Exponential and Log Functions

Graph: $y = 2^x$



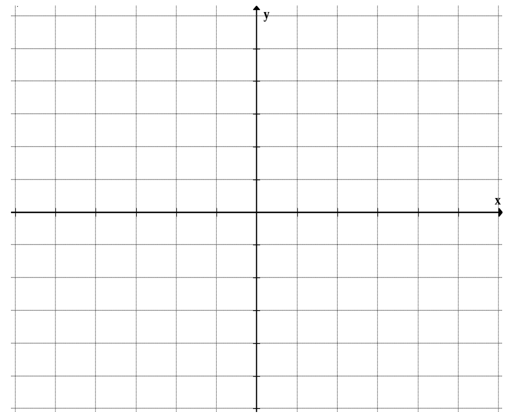
The domain is: ________________________.

The range is: ________________________.

Convert each from logarithmic form to exponential form (or vice versa):

Logarithmic Form	**Exponential Form**

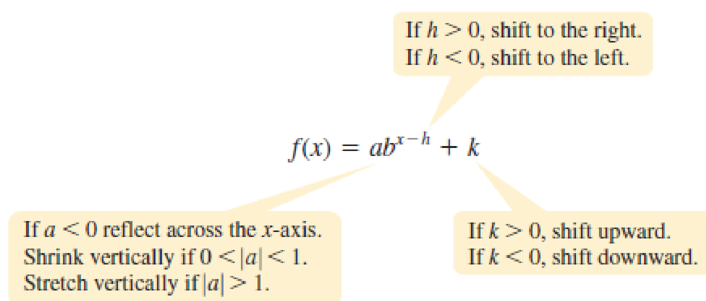
Graph $y = log_2 x$



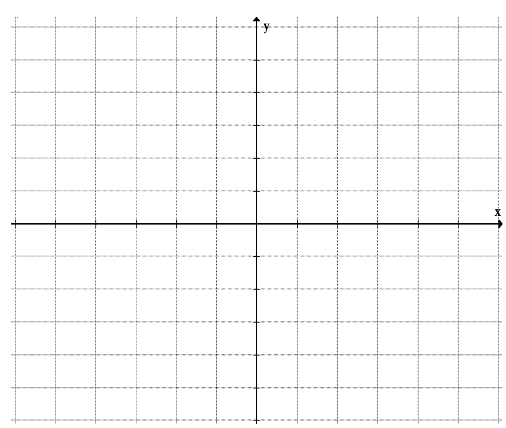
The domain is: ________________________.

The range is: ________________________.

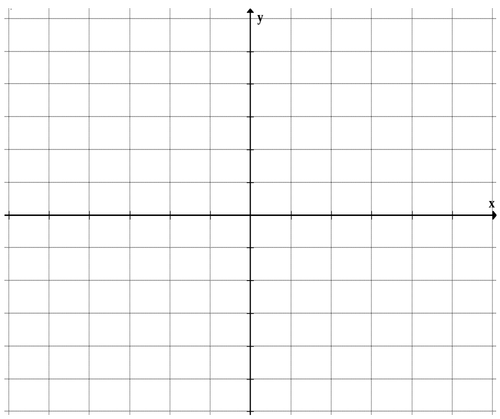
Transformations of exponential functions:



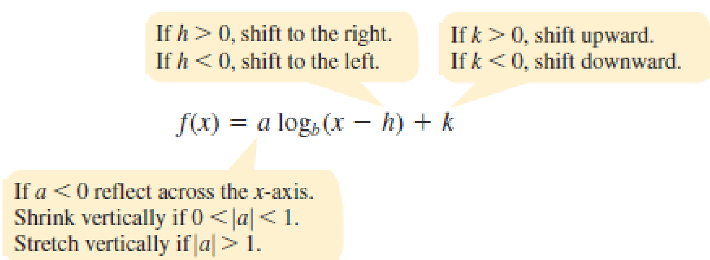
4.2 #26 Graph the function $g(x) = 4^x$ and write the domain and range in interval notation.



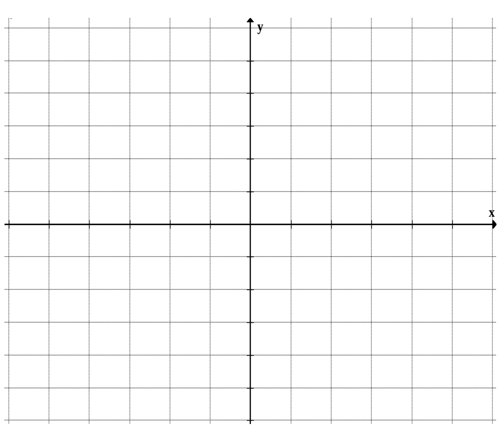
4.2 #38 Use the graph of $y = 4^x$ to graph the function $q(x) = 4^{x+1} + 2$. Write the domain and range in interval notation.



Transformations of logarithmic functions:



4.3 #80 a. Use transformations of the graph of $y = log_2 x$ to graph $y = log_2(x - 2) - 1$.



b. Write the domain and range in interval notation.

c. Determine the vertical asymptote.

In exercises 86 and 92, write the domain in interval notation.

4.3 #86 $k(x) = log_3(5x + 6)$

4.3 #90 $q(x) = log(x^2 + 10x + 9)$

C. Evaluating Exponential and Log Functions

Exponential keys on calculator:

Logarithmic keys on calculator:

On your calculator, find the following values:

1. $(3.07)^{1.42} \approx$

2. $e \approx$

3. $ln(317) \approx$

4. $e^{3.78} \approx$

In exercises 36-48, simplify the expression.

4.3 #36 $log_2 16$

4.3 #42 $log_3 \left(\frac{1}{9}\right)$

4.3 #48 $ln\left(\frac{1}{e^8}\right)$

Solve each equation:

1. $log_x 16 = 4$

2. $log_3 x = 5$

3. $log_5 25 = x$

4.3 #56 Estimate the value of each logarithm between two consecutive integers. Then use a calculator to approximate the value to 4 decimal places. For example, $\log 8970$ is between 3 and 4 because $10^3 < 8970 < 10^4$.

a. $\log 293{,}416$

b. $\log 897$

c. $\log 0.038$

d. $\log 0.00061$

e. $\log(9.1 \times 10^8)$

f. $\log(8.2 \times 10^{-2})$

Basic Properties of Logarithms

1. $log_b(1) = 0$
2. $log_b(b) = 1$
3. $log_b(b^x) = x$
4. $b^{log_b x} = x$

In exercises 59-68, simplify the expression.

4.3 #60 $log_6 6^7$

4.3 #64 $4^{log_4(a-c)}$

<u>Compound interest:</u> $A = P\left(1+\frac{r}{n}\right)^{nt}$

A represents the ___________________.

P represents the ___________________.

r represents the ___________________.

t represents the ___________________.

n represents the ___________________.

- Use n=____ for interest compunded annually.
- Use n=____ for interest compunded semi-annually.
- Use n=____ for interest compunded quarterly.
- Use n=____ for interest compunded monthly.
- Use n=____ for interest compunded daily.

For continuously compounding interest, use the formaula $A = Pe^{rt}$.

4.2 #55 Suppose that $10,000 is invested with 4% interest for 5 yr under the following compounding options. Complete the table.

	Compounding Option	n Value	Result
a.	Annually		
b.	Quarterly		
c.	Monthly		
d.	Daily		
e.	Continuously		

4.2 #66 The population of Canada in 2010 was approximately 34 million with an annual growth rate of 0.804%. At this rate, the population $P(t)$ (in millions) can be approximated by $P(t) = 34(1.00804)^t$, where t is the time in years since 2010.

a. Is the graph of P an increasing or decreasing exponential function?

b. Evaluate $P(0)$ and interpret its meaning in the context of the problem.

c. Evaluate $P(5)$ and interpret its meaning in the context of the problem. Round the population value to the nearest million.

d. Evaluate $P(15)$, $P(25)$, and $P(200)$.

4.4 – Properties of Logarithms

Properties of Logarithms:

1. $log(xy) = logx + logy$ product propery
2. $log\left(\frac{x}{y}\right) = logx - logy$ quotient property
3. $log(x^p) = p \cdot logx$ power property

4.4 #18 Use the product property of logarithms to write the logarithm as a sum of logarithms. Then simplify if possible.

$$log_7(49k)$$

4.4 #24 Use the quotient property of logarithms to write the logarithm as a sum of logarithms. Then simplify if possible.

$$log_9\left(\frac{m}{n}\right)$$

4.4 #28 Use the quotient property of logarithms to write the logarithm as a sum of logarithms. Then simplify if possible.

$$log\left(\frac{1000}{c^2+1}\right)$$

4.4 #34 Apply the power property of logarithms.

$$\ln(0.5)^{rt}$$

4.4 #42 Write the logarithm as a sum or difference of logarithms. Simplify each term as much as possible.

$$\ln\left(\frac{\sqrt[4]{pq}}{t^3 m}\right)$$

4.4 #62 Write the logarithmic expression as a single logarithm with coefficient 1, and simplify as much as possible.

$$15\log c - \frac{1}{4}\log d - \frac{3}{4}\log k$$

4.4 #62 Write the logarithmic expression as a single logarithm with coefficient 1, and simplify as much as possible.

$log(9t^3 - 5t) + logt^{-1}$

4.4 #74 Use $log_b 2 \approx 0.356$, $log_b 3 \approx 0.565$, and $log_b 5 \approx 0.827$ to approximate the value of the given logarithm.

$$log_b 12$$

Change-of-Base Formula

Let a and b be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then for any positive real number x,

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Note: The change-of-base formula converts a logarithm of one base to a ratio of logarithms of a different base. For the purpose of using a calculator, we often apply the change-of-base formula with base 10 or base e.

$$\log_b x = \frac{\log x}{\log b}$$

Original base is b. Ratio of base 10 logarithms

$$\log_b x = \frac{\ln x}{\ln b}$$

Original base is b. Ratio of base e logarithms

4.4 #80 a. Estimate the value of $log_3 15$ between two consecutive integers. For example, $log_2 7$ is between 2 and 3 because $2^2 < 7 < 2^3$.

b. Use the change-of-base formula and a calculator to approximate the logarithm to 4 decimal places.

c. Check the result by using the related exponential equation.

4.5 – Exponential and Logarithmic Equations

4.5 #16 Solve the equation.

$$5^{2z+2} = 625$$

4.5 #18 Solve the equation.

$$7^{2x-3} = \left(\frac{1}{49}\right)^{x+1}$$

For exercises 24-36, solve the equation. Write the solution set with the exact values given in terms of common or natural logarithms. Also give approximate solutions to 4 decimal places.

4.5 #24 $2^z = 70$

For exercises 24-36, solve the equation. Write the solution set with the exact values given in terms of common or natural logarithms. Also give approximate solutions to 4 decimal places.

4.5 #28 $10^{5+8x} + 4200 = 84{,}000$

4.5 #36 $e^{2x} - 6e^{x} - 16 = 0$

For exercises 42-60, solve the equation. Write the solution set with the exact values. Also give approximate solutions to 4 decimal places, if necessary.

4.5 #42 $log_7(12 - t) = log_7(t + 6)$

4.5 #46 $5log_6(7w + 1) = 10$

For exercises 42-60, solve the equation. Write the solution set with the exact values. Also give approximate solutions to 4 decimal places, if necessary.

4.5 #50 $\log(q - 6) = 3.5$

4.5 #56 $log_4(5x - 13) = 1 + log_4(x - 2)$

4.5 #58 $log_2 x = 4 - log_2(x - 6)$

4.5 #60 $\ln x + \ln(x - 3) = \ln(5x - 7)$

4.5 #66 Use the model $A = P\left(1 + \frac{r}{n}\right)^{nt}$. Barb puts aside $10,000 in an account with an interest reinvested monthly at 2.5%. How long will it take for her to *earn* $2000? Round to the nearest month.

4.6 – Modeling with Exponential and Logarithmic Functions

4.6 #14 Solve for the indicated variable.

$N = N_0 e^{-0.025t}$ for t (used in chemistry)

4.6 #18 Solve for the indicated variable.

$L = 10\log\left(\frac{I}{I_0}\right)$ for I (used in medicine)

4.6 #24 Suppose that \$50,000 from a retirement account is invested in a large cap stock fund. After 20 yr, the value is \$194,809.67.

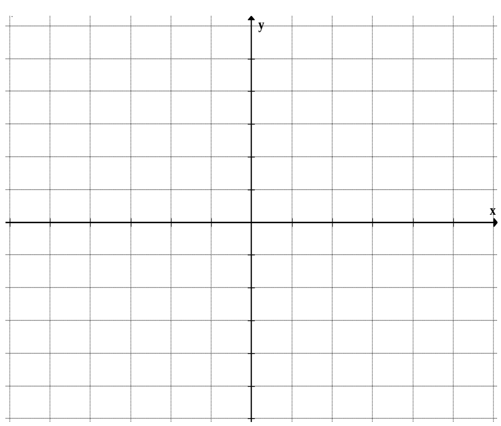
a. Use the model $A = Pe^{rt}$ to determine the average rate of return under continuous compounding.

b. How long will it take the investment to reach one-quarter million dollars? Round to 1 decimal place.

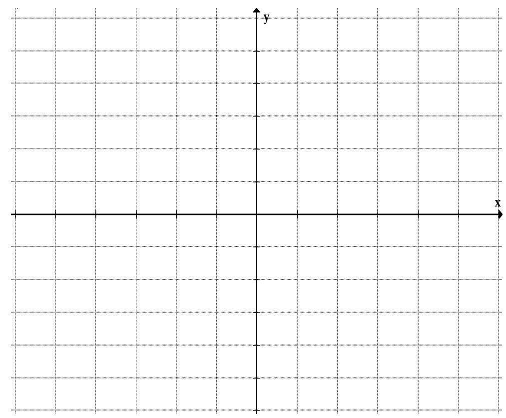
Some Chapter 4 Review Problems

1. Solve: $-log_2(x-4) = 3 - log_2(x+3)$

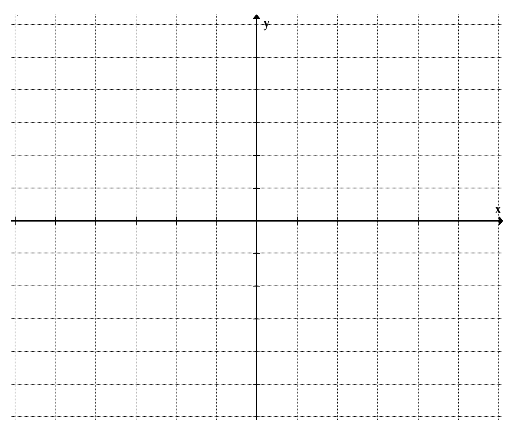
2. Graph: $g(x) = 3log_4(x-2) + 1$



3. Graph: $h(x) = -2log_2(x+1) + 3$



4. Graph $y = 3^{x+2} - 1$



5. Solve: $3^{x+4} = 81$

6. Solve: $3^{x+4} = 85$

7. Solve: $log x - 1 = -log(x - 9)$

8. Solve: $\ln(21) = 1 + \ln(x - 2)$

9. For continuously compounding interest, at what interest rate should \$500 be invested so that is grows to \$750 in 8 years?

10. For how long should \$800 be invested at 4.3%, compounded daily, in order for it to grow to \$2000?

11. For $f(x)=\dfrac{7x-9}{-8x+5}$, find $f^{-1}(x)$, and the domain and range of $f^{-1}(x)$.