

# Lecture Guide

## Math 105 - College Algebra

### Chapter 1

to accompany

“College Algebra” by Julie Miller

Corresponding Lecture Videos can be found at



Prepared by

**Stephen Toner & Nichole DuBal**

Victor Valley College

Last updated: 2/16/13

## Chapter R – Review Topics

### R.1 Interval Notation

Inequality      Graph      Interval Notation

### R.1 Set-Builder Notation

R.1 #56 Write the interval notation and set-builder notation for the given graph.



### Finding Domains of Functions

The **domain** of an expression (or function) is the set of values which satisfy the expression.

To find the domain, start with a "default"

domain of \_\_\_\_\_

and then take away values which create ...

\*\*

\*\*

\*\*

\*Find the domain of each function:

$$f(x) = \frac{x + 2}{x - 3}$$

$$g(x) = \sqrt{x - 3}$$

$$h(x) = \log_w(x - 3)$$

$$f(x) = \frac{x + 7}{x^2 - x - 6}$$

## 1.1 – Linear Equations and Rational Equations

### A. Linear Equations

1.1 #20 Solve the equation.

$$-5(u - 4) + 2 = 11 - (u - 3)$$

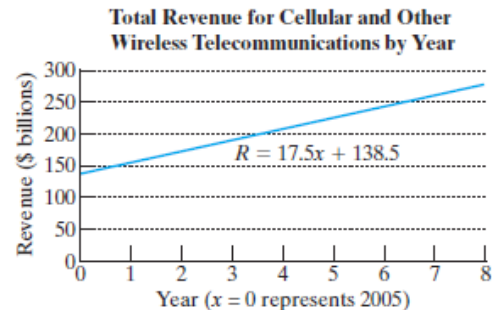
1.1 #34 Solve the equation.

$$\frac{t - 2}{3} - \frac{t + 7}{5} = \frac{t - 4}{10} + 2$$

1.1 #38 The total revenue  $R$  (in \$ billions) for cellular and other wireless telecommunication industries in the United States can be modeled by  $R = 17.5x + 138.5$ , where  $x$  is the number of years since 2005.

a. Determine the revenue for the year 2007.

b. Determine the year in which the revenue was \$226 billion.



1.1 #44 Identify the equation as a conditional equation, a contradiction, or as an identity. Then give the solution set.

$$4(3 - 5n) + 1 = -4n - 8 - 16n$$

**B. Rational Equations**

1.1 #50 Determine the restrictions on  $x$ .

$$\frac{2}{x+1} - \frac{5}{x-7} = \frac{2}{3}$$

1.1 #54 Solve the equation.

$$\frac{1}{3} - \frac{4}{3t} = \frac{7}{t}$$

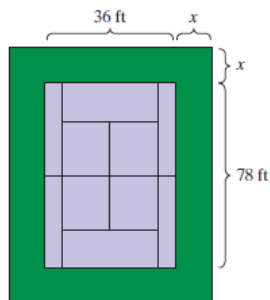
1.1 #64 Solve the equation.

$$\frac{4}{x^2 - 2x - 8} - \frac{1}{x^2 - 16} = \frac{2}{x^2 + 6x + 8}$$

1.1 #82 Solve  $S = \frac{n}{2}[2a + (n - 1)d]$  for  $a$ .

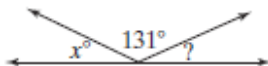
## 1.2 – Applications with Linear Equations

1.2 #18 The Arthur Ashe Stadium tennis court is center court to the U.S. Open tennis tournament. The dimensions of the court are 78 ft by 36 ft, with a uniform border of  $x$  feet around the outside for additional play area. If the perimeter of the entire play area is 396 ft, determine the value of  $x$ .



1.2 #30 Ebony bought a 5-yr Treasury note that paid the equivalent of 2.8% simple interest. She invested \$5000 more in a 10-yr bond earning 3.6% than she did in the Treasury note. If the total amount of interest from these investments was \$5300, determine the amount of principal for each investment.

1.2 #23 Write an expression in terms of  $x$  that represents the measure of the angle designated by the ? symbol.



1.2 #32 A nurse mixes 60 cc of a 50% saline solution with a 10% saline solution to produce a 25% saline solution. How much of the 10% solution should he use?

1.2 #36 Two planes leave from Atlanta, Georgia. One makes a 5.2-hr flight to Seattle, Washington, and the other makes a 2.5-hr flight to Boston, Massachusetts. The plane to Boston averages 44 mph slower than the plane to Seattle. If the total distance traveled by both planes is 3124 mi, determine the average speed of each plane.

1.2 #40 Tasha considers two sales jobs for different pharmaceutical companies. One pays a base salary of \$25,000 with a 16% commission on sales. The other pays \$30,000 with a 15% commission on sales.

a. Write a model representing the salary  $S_1$  (in \$) for the first job based on  $x$  dollars in sales.

b. Write a model representing the salary  $S_2$  (in \$) for the second job based on  $x$  dollars in sales.

c. For how much in sales will the two jobs result in equal salaries?

1.2 #44 Marta can vacuum the house in 40 min. It takes her daughter 1 hr to vacuum the house. How long would it take them if they worked together?

1.2 #52 To estimate the number of bass in a lake, a biologist catches and tags 24 bass. Several weeks later, the biologist catches a new sample of 40 bass and finds that 4 are tagged. How many bass are in the lake?

**1.3 – Complex Numbers**1.3 #18 Write in terms of  $i$  and simplify.

$$\sqrt{-100}$$

1.3 #20 Write in terms of  $i$  and simplify.

$$\sqrt{-63}$$

1.3 #30 Write in terms of  $i$  and simplify.

$$\sqrt{-10}\sqrt{-15}$$

1.3 #46 Simplify and write the result in standard form,  $a + bi$ .

$$-4$$

1.3 #54 Simplify and write the result in standard form,  $a + bi$ .

$$\frac{-10 + \sqrt{-125}}{5}$$

1.3 #56 Simplify the powers of  $i$ .

a.  $i^{32}$     b.  $i^{47}$     c.  $i^{66}$     d.  $i^{-27}$

1.3 #64 Perform the indicated operation.  
Write the answer in standard form,  $a + bi$ .

$$\left(\frac{3}{5} - \frac{1}{8}i\right) - \left(\frac{7}{10} + \frac{1}{6}i\right)$$

1.3 #74 Perform the indicated operation.  
Write the answer in standard form,  $a + bi$ .

$$(2 - 5i)(8 + 2i)$$

1.3 #84 For the given number, (a) identify the complex conjugate, and (b) determine the product of the number and its conjugate.

$$4 - 5i$$

1.3 #96 Perform the indicated operation.  
Write the answer in standard form,  $a + bi$ .

$$\frac{10 - 3i}{11 + 4i}$$

## 1.4 – Quadratic Equations

Summary:  $ax^2 + bx + c = 0$

a. If  $c = 0$ , use factoring.

b. If  $b = 0$ , square root each side.

c. If  $a = 0$ , it's not quadratic!

Solve  $x^2 - 2x - 15 = 0$  in three ways.

### Zero Product Property

If  $mn = 0$ , then  $m = 0$  or  $n = 0$ .

Verbal Explanation

If the product of two factors is zero, then at least one factor is zero.

Example

$(x + 4)(x - 2) = 0$  implies that  $x + 4 = 0$  or  $x - 2 = 0$ , indicating that  $x = -4$  or  $x = 2$ .

1.4 #18 Solve by applying the zero product property.  $6m(m + 4) = m - 15$

### Square Root Property

If  $x^2 = k$ , then  $x = \pm\sqrt{k}$ .

The solution set is  $\{\sqrt{k}, -\sqrt{k}\}$  or more concisely  $\{\pm\sqrt{k}\}$ .

1.4 #34 Solve by using the square root property.  $\left(a - \frac{1}{3}\right)^2 = -\frac{47}{9}$

1.4 #42 Determine the value of  $n$  that makes the polynomial a perfect square trinomial. Then factor as the square of a binomial.

$$k^2 + \frac{2}{5}k + n$$



1.4 #44 Solve by completing the square and applying the square root property.

$$x^2 + 14x - 3 = 0$$

1.4 #52 Solve by completing the square and applying the square root property.

$$-2x^2 - 14x + 5 = 0$$

1.4 #62 Solve by using the quadratic formula.

$$m(m + 10) = -34$$

1.4 #70 Solve by using the quadratic formula.

$$\frac{1}{3}x^2 - \frac{7}{6} = \frac{3}{2}x$$

The  $b^2 - 4ac$  expression found in the quadratic formula is called the \_\_\_\_\_.

3 Cases for the Discriminant:

1. If  $b^2 - 4ac$  is negative, the quadratic equation has \_\_\_\_\_.
2. If  $b^2 - 4ac$  is zero, the quadratic equation has \_\_\_\_\_.
3. If  $b^2 - 4ac$  is positive, the quadratic equation has \_\_\_\_\_.

1.4 #100 (a) Evaluate the discriminant, and (b) determine the number and type of solutions to the equation.

$$5x^2 - 2x + 4 = 0$$

1.4 #116 Solve  $dy^2 + my = p$  for  $y$ .

1.4 #127 Write an equation with integer coefficients and the variable  $x$  that has the solution set  $\{4, -2\}$ .

1.4 #135 Write an equation with integer coefficients and the variable  $x$  that has the solution set  $\{1 \pm 2i\}$ .

### Problem Recognition Exercises

Identify the statement as an expression or as an equation. Then simplify the expression or solve the equation.

1.PRE #1

a.  $(2x - 5)(3x + 1)$

b.  $(2x - 5)(3x + 1) = 0$

1.PRE #8

a.  $\frac{x}{x-2} + \frac{2}{3} = \frac{2}{x-2}$

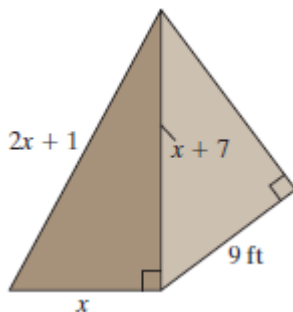
b.  $\frac{x}{x-2} + \frac{2}{3} - \frac{2}{x-2}$

## 1.5 – Applications of Quadratic Equations

1.5 #20 a. Write an equation representing the fact that the sum of the squares of two consecutive integers is 181.

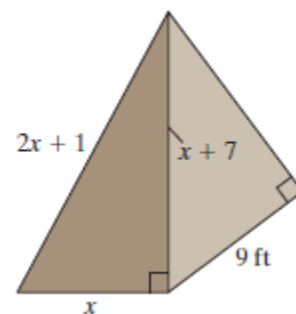
b. Solve the equation from part (a) to find the two integers.

1.5 #30 The portion of a roof truss is given in the figure. The triangle on the left is configured such that the longer leg is 7 ft longer than the shorter leg, and the hypotenuse is 1 ft more than twice the shorter leg.



a. Find the lengths of the sides of the triangle on the left.

b. Find the lengths of the sides of the triangle on the right.



1.5 #36 The gas mileage for a certain vehicle can be approximated by  $m = -0.04x^2 + 3.6x - 49$ , where  $x$  is the speed of the vehicle in mph. Determine the speed(s) at which the car gets 30 mpg. Round to the nearest mph.

1.5 #42 In a classic *Seinfeld* episode, Jerry tosses a loaf of bread (a marble rye) straight upward to his friend George who is leaning out of a third-story window.

a. If the loaf of bread leaves Jerry's hand at a height of 1 m with an initial velocity of 18 m/sec, write an equation for the vertical position of the bread  $s$  (in meters)  $t$  seconds after release. Use the model

$$s = -\frac{1}{2}gt^2 + v_0t + s_0.$$

b. How long will it take the bread to reach George if he catches the bread on the way up at a height of 16 m? Round to the nearest tenth of a second.

## 1.6 – More Equations and Applications

### A. Polynomial Equations

1.6 #12 Solve the equation.

$$98t^3 - 49t^2 - 8t + 4 = 0$$

### B. Rational Equations

1.6 #20 Solve the equation.

$$\frac{4c}{c-5} - \frac{1}{c+1} = \frac{3c^2+3}{c^2-4c-5}$$

**C. Radical Equations and Rational Exponents**

1.6 #34 Solve the equation.

$$\sqrt{2n + 29} + 3 = n$$

1.6 #40 Solve the equation.

$$\sqrt{d + 4} - \sqrt{6 + 2d} = -1$$

1.6 #46 Solve the equation.

$$4(y - 3)^{3/4} = 20$$

**D. Equations In Quadratic Form**

1.6 #56 Make an appropriate substitution and solve the equation.

$$(y^2 - 3)^2 - 9(y^2 - 3) - 52 = 0$$

1.6 #60 Make an appropriate substitution and solve the equation.

$$3d^{2/3} - d^{1/3} - 4 = 0$$

1.6 #67 Solve  $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$  for  $p$ .

**1.7 – Linear and Compound Inequalities**

1.7 #14 Solve the inequality. Graph the solution set, and write the solution set in set-builder notation and interval notation.

$$-8t + 1 < 17$$

1.7 #22 Solve the inequality. Graph the solution set, and write the solution set in set-builder notation and interval notation.

$$\frac{y + 3}{4} - \frac{3y + 1}{6} > -\frac{1}{12}$$

1.7 #26 Solve the inequality. Graph the solution set, and write the solution set in set-builder notation and interval notation.

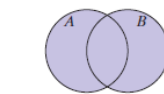
$$2(3x + 1) - 4x > 2(x + 8) - 5$$

1.7 #30 Solve the inequality. Graph the solution set, and write the solution set in set-builder notation and interval notation.

$$2x - 9 < 6(x - 1) - 4x$$

**Union and Intersection of Sets**

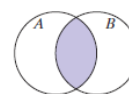
The union of sets  $A$  and  $B$ , denoted  $A \cup B$ , is the set of elements that belong to set  $A$  or to set  $B$  or to both sets  $A$  and  $B$ . See Figure 1-3.



$A \cup B$   
A union B  
The elements in A or B or both

Figure 1-3

The intersection of sets  $A$  and  $B$ , denoted  $A \cap B$ , is the set of elements common to both set  $A$  and set  $B$ . See Figure 1-4.



$A \cap B$   
A intersection B  
The elements common to A and B

Figure 1-4

1.7 #32 Find the union or intersection as indicated, given the following sets:

$$X = \{-10, -9, -8, -7\},$$

$$Y = \{-10, -8, -6, -4\},$$

$$Z = \{-6, -5, -4, -3, -2\}$$

a.  $X \cup Y$

b.  $X \cap Y$

c.  $X \cup Z$

d.  $X \cap Z$

e.  $Y \cup Z$

f.  $Y \cap Z$

Example of intersections and unions of intervals:



1.7 #34 Find the union or intersection as indicated, given the following sets:

$$M = \{y | y \geq -3\},$$

$$N = \{y | y \geq 5\},$$

$$P = \{y | y < 0\}$$

First, graph  $M$ ,  $N$  and  $P$  individually.

a.  $M \cup N$

b.  $M \cap N$

c.  $M \cup P$

d.  $M \cap P$

e.  $N \cup P$

f.  $N \cap P$

1.7 #42 Solve the compound inequality. Graph the solution set, and write the solution set in interval notation.

a.  $n - 6 > 1$  or  $\frac{3}{4}n \geq 6$

b.  $n - 6 > 1$  and  $\frac{3}{4}n \geq 6$

1.7 #50 Solve the compound inequality. Graph the solution set, and write the solution set in interval notation.

$$-6 \leq -3x + 9 < 0$$

1.7 #60 A 10-yr-old competes in gymnastics. For several competitions she received the following "All-Around" scores: 36, 36.9, 37.1, and 37.4. Her coach recommends that gymnasts whose "All-Around" scores average at least 37 move up to the next level. What "All Around" scores in the next competition would result in the child being eligible to move up?

1.7 #77 Determine the set of values for  $x$  for which the radical expression would produce a real number. For example, the expression  $\sqrt{x-1}$  is a real number if  $x-1 \geq 0$  or equivalently,  $x \geq 1$ .

a.  $\sqrt{x+4}$

b.  $\sqrt[3]{x+4}$

1.7 #93 Solve the inequality. Write the solution set in interval notation.

$$-11 < 6y + 7 \text{ and } 6y + 7 < -5$$



**1.8 – Absolute Value Equations & Inequalities**

Properties:

1.  $|x| = k \rightarrow$
2.  $|x| < k \rightarrow$
3.  $|x| > k \rightarrow$

*1.8 #14* Solve the equations.

- a.  $|w| = 2$
  
- b.  $|w| = 0$
  
- c.  $|w| = -2$

*1.8 #18* Solve the equation.

$$4|2t + 7| + 2 = 22$$

*1.8 #22* Solve the equation.

$$6 = 7 + |9z - 3|$$

*1.8 #24* Solve the equation.

$$\left|2 - \frac{1}{3}p\right| - \frac{7}{6} = \frac{1}{2}$$

*1.8 #26* Solve the equation.

$$|2a - 3| = |a + 2|$$

*1.8 #34* Solve the equation or inequality. Write the solution set to each inequality in interval notation.

- a.  $|y| = 8$
  
- b.  $|y| < 8$
  
- c.  $|y| > 8$

1.8 #38 Solve the inequality and write the solution set in interval notation.

$$2|7 - y| + 1 < 17$$

1.8 #44 Solve the inequality and write the solution set in interval notation.

$$15 < [-2d - 3] + 6$$

1.8 #56 Write the solution set.

a.  $|c - 3| = 0$

b.  $|c - 3| < 0$

c.  $|c - 3| \leq 0$

d.  $|c - 3| > 0$

e.  $|c - 3| \geq 0$

1.8 #64 Write an inequality that represents the statement. "The distance between  $y$  and  $L$  is less than  $\varepsilon$ " ( $\varepsilon$  is the Greek letter "epsilon").

1.8 #66 A box of cereal is labeled to contain 16 oz. A consumer group takes a sample of 50 boxes and measures the contents of each box. The individual content of each box differs slightly from 16 oz, but by no more than 0.5 oz.

a. If  $x$  represents the exact weight of the contents of a box of cereal, write an absolute value inequality that represents an interval in which to estimate  $x$ .

b. Solve the inequality and interpret the answer.