

rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
deg	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

Pythagorean Identities

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$1 + \tan^2\alpha = \sec^2\alpha$$

$$1 + \cot^2\alpha = \csc^2\alpha$$

Sum & Difference

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$$

Double-Angles

$$\sin(2\alpha) = 2\sin\alpha \cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$\cos(2\alpha) = 2\cos^2\alpha - 1$$

$$\cos(2\alpha) = 1 - 2\sin^2\alpha$$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

Half-Angles

$$\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos\alpha}{2}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 + \cos\alpha}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1 - \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 + \cos\alpha}$$

Product-To-Sum

$$\sin\alpha \sin\beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos\alpha \cos\beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin\alpha \cos\beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos\alpha \sin\beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum-to-Product

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Power-Reducing

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Special limits

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right) = 1$$

$$\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x}\right) = 0$$

Heron's Area Formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

Area of Oblique Triangles

$$\text{Area} = \frac{1}{2} bc \sin A$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$\text{Area} = \frac{1}{2} ac \sin B$$

Line Formulas

$$y - y_1 = m(x - x_1) \quad \text{point-slope}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta$$

$$\text{dist. from } (x_1, y_1) \text{ to } Ax + By = C$$

$$\text{is given by } d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Conic Sections

$$(x - h)^2 = 4p(y - k)$$

$$\frac{(x - h)^2}{a^2} \pm \frac{(y - k)^2}{b^2} = 1$$

$$\text{asymptotes: } y = \pm(x - h) + k$$

Sequences & Series

$$a_n = a_1 + (n - 1)d$$

$$a_n = a_1 r^{n-1} \quad S_\infty = \frac{a_1}{1 - r}$$

$$S_n = \frac{a_1 - a_n r}{1 - r} = \frac{a_1 - a_1 r^n}{1 - r}$$

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n - 1)d]$$

Differentiation Rules

$$\frac{d}{dx}[c] = 0 \quad \frac{d}{dx}[x] = 1$$

$$\frac{d}{dx}[cu] = cu'$$

$$\frac{d}{dx}[u \pm v] = u' \pm v'$$

$$\frac{d}{dx}[fgh] = f'gh + fg'h + fgh'$$

$$\frac{d}{dx}[uv] = uv' + vu'$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$$

$$\frac{d}{dx}[u^n] = nu^{n-1}u'$$

$$\frac{d}{dx}[|u|] = \frac{u'}{|u|}(u'),$$

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

$$\frac{d}{dx}[e^u] = e^u u'$$

$$\frac{d}{dx}[\sin u] = (\cos u)u'$$

$$\frac{d}{dx}[\cos u] = -(\sin u)u'$$

$$\frac{d}{dx}[\tan u] = (\sec^2 u)u'$$

$$\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$$

$$\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$$

$$\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$$

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arc cot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arc sec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}[\operatorname{arc csc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Trig Graphing

$$y = a \sin(bx + c) + d$$

amplitude: $|a|$

period: $\frac{2\pi}{|b|}$

phase sht: $-\frac{c}{b}$

vertical anslation: d

Integration Formulas

$$\int du = u + C$$

$$\int kf(u) du = k \int f(u) du$$

$$\int [f(u) \pm g(u)] du = \int f(u) du \pm \int g(u) du$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int \frac{du}{u \ln u} = \ln|\ln u| + C$$

$$\int \ln u du = u \ln u - u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int e^u du = e^u + C$$

$$\int ue^u du = e^u(u-1) + C$$

$$\int \sin u du = -\cos u + C$$

$$\int u \sin u du = \sin u - u \cos u + C$$

$$\int \sin^2 u du = \frac{1}{2}u - \frac{1}{4}\sin(2u) + C$$

$$\int \cos u du = \sin u + C$$

$$\int u \cos u du = \cos u + u \sin u + C$$

$$\int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\sin(2u) + C$$

$$\int \tan u du = -\ln|\cos u| + C$$

$$\int \tan^2 u du = \tan u - u + C$$

$$\int \cot u du = \ln|\sin u| + C$$

$$\int \cot^2 u du = -\cot u - u + C$$

$$\int \sec u du = \ln|\sec u + \tan u| + C$$

$$\int \csc u du = -\ln|\csc u + \cot u| + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arc sec} \frac{|u|}{a} + C$$

Complex Numbers

$$a + bi = (r \cos \vartheta) + (r \sin \vartheta)i$$

If $z = a + bi$,

$$\text{then } |z| = |a + bi| = \sqrt{a^2 + b^2}$$

$$\text{If } z_1 = r_1(\cos \vartheta_1 + i \sin \vartheta_1)$$

$$\text{and } z_2 = r_2(\cos \vartheta_2 + i \sin \vartheta_2)$$

then

$$z_1 z_2 = r_1 r_2 [\cos(\vartheta_1 + \vartheta_2) + i \sin(\vartheta_1 + \vartheta_2)]$$

and

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\vartheta_1 - \vartheta_2) + i \sin(\vartheta_1 - \vartheta_2)]$$

Log Properties

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \approx 2.71828$$

$$\log_a a = 1 \quad \log_a 1 = 0$$

$$\log_a (a^x) = x \quad a^{\log_a x} = x$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\log_a (uv) = \log_a u + \log_a v$$

$$\log_a \left(\frac{u}{v}\right) = \log_a u - \log_a v$$

$$\log_a (u^n) = n \log_a u$$

Vectors

dot product: $u \cdot v = u_1 v_1 + u_2 v_2$

If $v = \langle v_1, v_2 \rangle$, then $\|v\| = \sqrt{v_1^2 + v_2^2}$

$$\cos \vartheta = \frac{u \cdot v}{\|u\| \cdot \|v\|}$$

$$\|u\| \cdot \|v\| \cos \vartheta = u \cdot v$$

$$v = \|v\| \langle \cos \vartheta, \sin \vartheta \rangle$$

$$= \|v\| (\cos \vartheta)i + \|v\| (\sin \vartheta)j$$

, where $i = \langle 1, 0 \rangle$ and $j = \langle 0, 1 \rangle$.

De Moivre's Theorem

If $z = r(\cos \vartheta + i \sin \vartheta)$ is complex and n is a natural number, then

$$z^n = r^n (\cos n\vartheta + i \sin n\vartheta).$$

Complex number

$z = r(\cos \vartheta + i \sin \vartheta)$ has exactly n roots given by:

$$\sqrt[n]{r} \left(\cos \left(\frac{\vartheta + 2\pi k}{n} \right) + i \sin \left(\frac{\vartheta + 2\pi k}{n} \right) \right),$$

where $k=0, 1, 2, \dots, n-1$.